

VALIDATION CASE 1:

Finite Element with modal reduction

a) Describe how was the model built:

(Number of nodes and elements, types of elements, mean size of the elements, frequency range, used parameters, degree of freedom, Is used only one mesh for all the interval?, Are taken particular countermeasure?,... Add one or two plot of the mesh.)

The model was originally retrieved from the Mid-Frequency website including: Meshes for the acoustic cavity (62561 tetra elements) and the plate (16*34 quad elements), "Mass" and "Stiffness" sparse matrices for the plate (6*495 dofs after BC) and acoustic domain (11758 dofs).

The acoustic cavity mesh was however recreated with Nastran, exported to Matlab, to satisfy the following points:

- Have a compatible quad mesh at the structural-acoustic interface to enable a simple implementation of coupling conditions in Matlab,
- Have a quad mesh node distribution at the acoustic-impedance boundary surface to enable a simple integral implementation of the impedance boundary conditions,
- Refine the acoustic mesh to be suitable up to 500 Hz.

The final mesh includes:

- 124530 linear tetrahedron acoustic elements (mean size 0.09 m), i.e. approximately 8 elements / wavelength at 500 Hz, corresponding to 22983 nodes (and acoustic dofs),
- 16*34 linear quad shell elements (mean size 0.05 m), i.e. approximately 1 element / $\frac{1}{4}$ wavelength at 500 Hz.

Coupling quad surface integral elements were implemented in Matlab to account for the impedance boundary conditions and the structural-acoustic coupling.

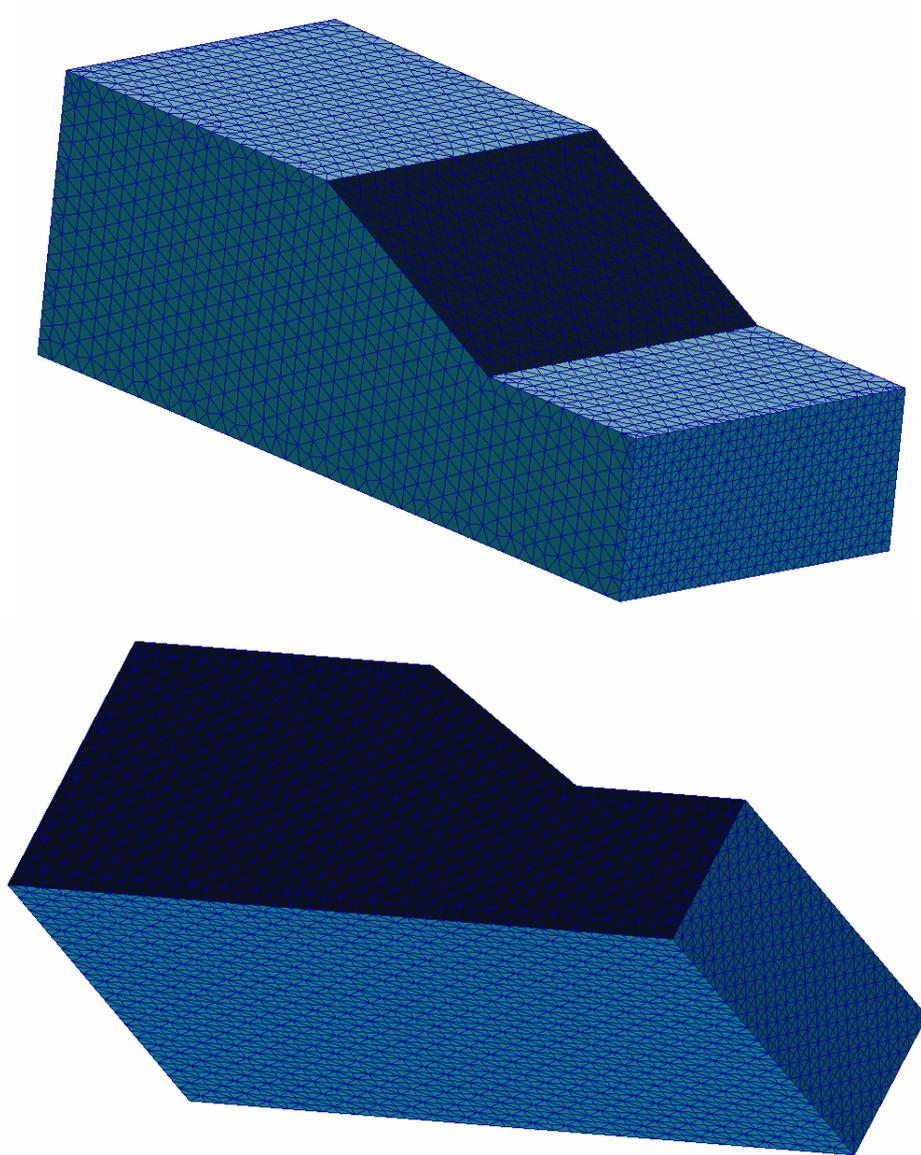


Figure 1: Mesh of the problem.

The material parameters used are those proposed in the D8 academic benchmark report.

The complete coupled Finite Element problem was built in Matlab. Solutions of the complete problem are given. Then a component mode synthesis approach is tested to reduce the acoustic domain, or both the acoustic and structural subdomains. Interface dofs are chosen among acoustic dofs.

b) Something about the computation:

(Total time, relation between time and frequency, number of CPU, software used, Is used some special computational tool?,...)

From the FE implementation in Matlab, and computation run on a single CPU Intel T9500 at 2.6 GHz, with 3 Gb of memory, in a Linux environment.

The overall computational times are given for the entire frequency range (1-500 Hz) using direct computations at 1 Hz intervals.

CPU time for the complete FE model: 15600s (4h20min)

The reduced models were built with dynamic condensation of internal dofs to interface dofs. "Interface" dofs include: acoustic dofs at interface with the impedance boundary condition and with the structural domain, as well as the output acoustic dof. In case of dynamic condensation of the structural domain, the excitation dof is also included into the interface set.

CPU time for the acoustic-reduced CMS approach – 1060 Hz truncation:

- Computation of 840 acoustic modes (1060 Hz truncation): 714s (11min54s),
- Computation of 1931 attachment functions (impedance and structure interfaces, acoustic output point): 222s (3min42s),
- Computation of frequency response: 3775s (1h2min55s).

Overall CPU time: 4711s (1h18min31s) – 3.3 times as fast as the complete FE model.

CPU time for the acoustic-reduced CMS approach – 1250 Hz truncation:

- Computation of 1253 acoustic modes (1250 Hz truncation): 1460s (24min20s),
- Computation of 1931 attachment functions (impedance and structure interfaces, acoustic output point): 221s (3min41s),
- Computation of frequency response: 3931s (1h5min31s).

Overall CPU time: 5612s (1h33min32s) – 2.8 times as fast as the complete FE model.

Of course, the CPU time can be slightly enhanced if a resolution per frequency band is considered. Thus, building an adaptive reduced model, best suited for each frequency band, allows to start the resolution with fewer normal modes in the transformation basis, therefore improving the efficiency of the dynamic condensation step.

Non significant CPU time improvements were observed with dynamic condensation of the structural part considering its small size.

c) Results:

(Add chart of the results and write comments)

1. Conservative and damped problems.

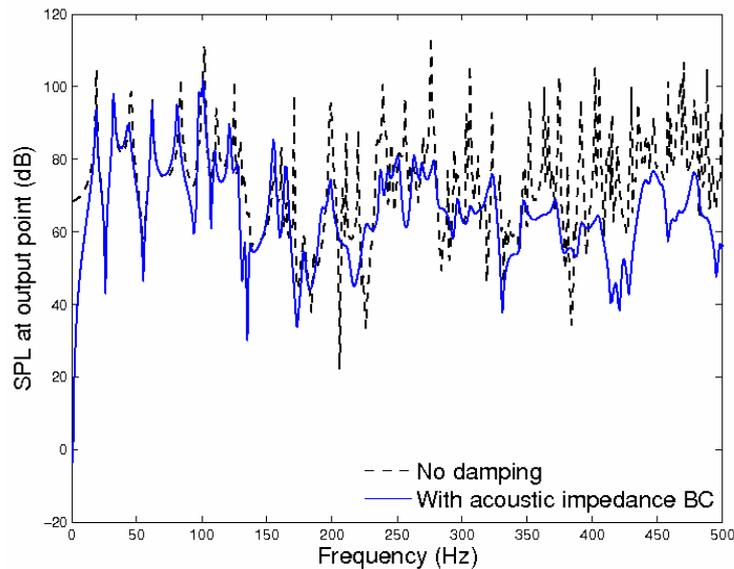


Figure 2: SPL FRF at output point:
Effect of the impedance boundary condition.

Figure 2 illustrates the reference solution for the undamped and melanine-damped acoustic cavity. It shows the SPL reduction with the adjunction of a sound absorbing layer, modelled with an impedance boundary condition. As expected, the impact of damping increases with the frequency.

The output point in the cavity is taken at point (0.534,0.403,0.495). The suggested point in the D8 academic benchmark report was (0.51,0.425,0.5), as the position of a microphone. This point was not exactly set in the mesh, and comments are made on this aspect in the next section.

2. Questioning the single output point results.

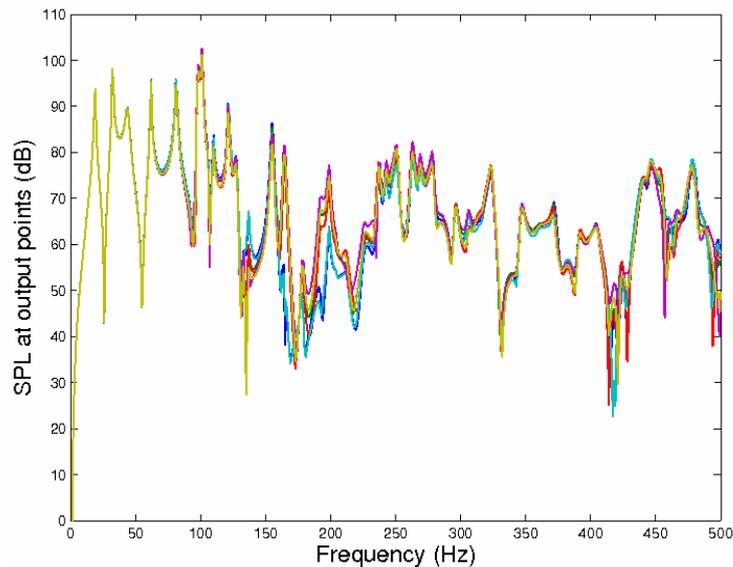


Figure 3: SPL FRF at 6 different output points
in a cubic zone of $10*10*10 \text{ cm}^3$.

The use of a single point as an output is not very well suited for an analysis out of the very low frequency region. As illustrated on Figure 3, the response becomes very sensitive to small variations of the parameters with increasing frequencies (here illustrated with small variations of the measurement position). A deterministic approach is therefore not the most relevant. At minimum, an output averaged over a significant volume would have been more sensible.

The frequency responses given in the following are however given at the output point (0.534,0.403,0.495), in order to match the validation case for numerical techniques.

3. Dynamic condensation of the acoustic component.

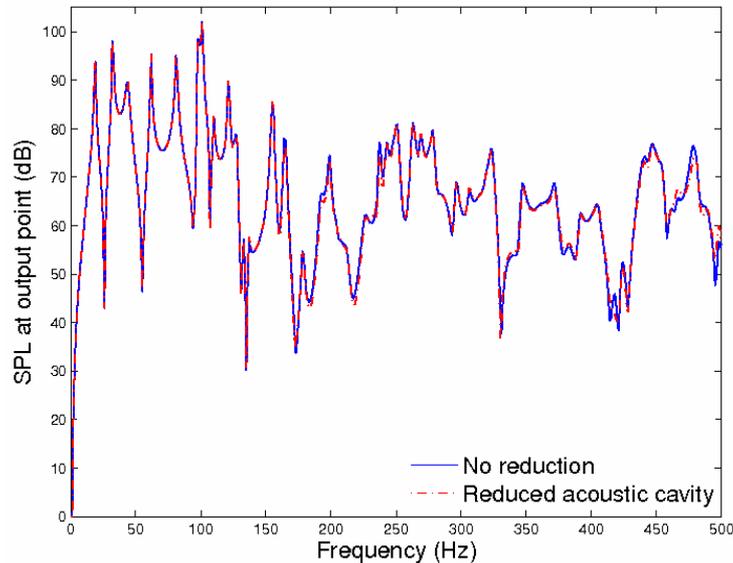


Figure 4: SPL FRF at output point:
Modal reduction of acoustic domain (1060 Hz truncation).

Figure 4 and Figure 5 present the sound pressure level frequency responses at the output point, comparing the precision reached by a modal-based reduced model to the reference finite element solution.

The acoustic cavity is dynamically condensed on its interface degrees of freedom with the structural domain and the impedance boundary surface. The output dof is kept uncondensed so that no back-substitution step is required.

Figure 4 presents the results for a truncation at 1060 Hz (slightly more than twice the highest frequency of interest) for the internal acoustic modal basis (interface degrees of freedom restrained). The corresponding computational time is overall 3.3 times as fast as the resolution of the complete model (4.1 times as fast if only the resolution time is accounted for), giving satisfactory approximation of the reference solution.

Figure 5 presents the results for a truncation at 1250 Hz (2.5 times the highest frequency of interest) for the internal acoustic modal basis (interface degrees of freedom restrained). The corresponding computational time is overall 2.8 times as fast as the resolution of the complete model (3.95 times as fast if only the resolution time is accounted for), bringing slight corrections to the approximation above 350 Hz.

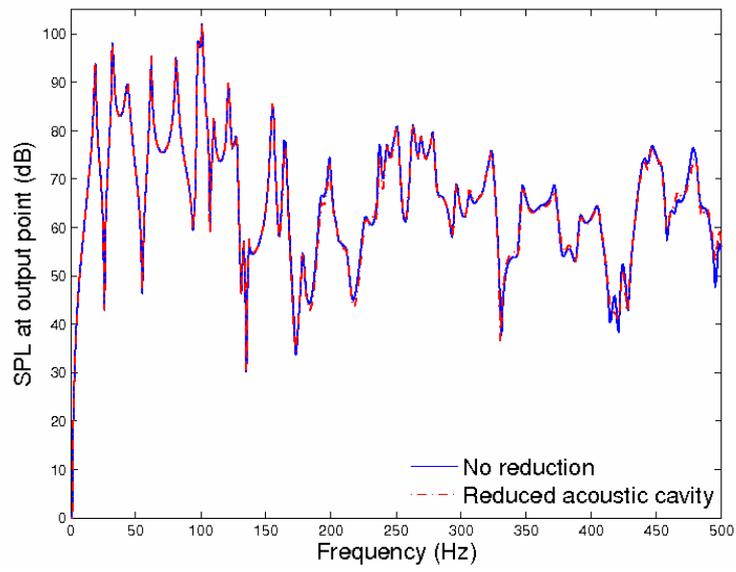


Figure 5: SPL FRF at output point:
Modal reduction of acoustic domain (1250 Hz truncation).

4. Dynamic condensation of the acoustic and structural components on acoustic interface dofs.

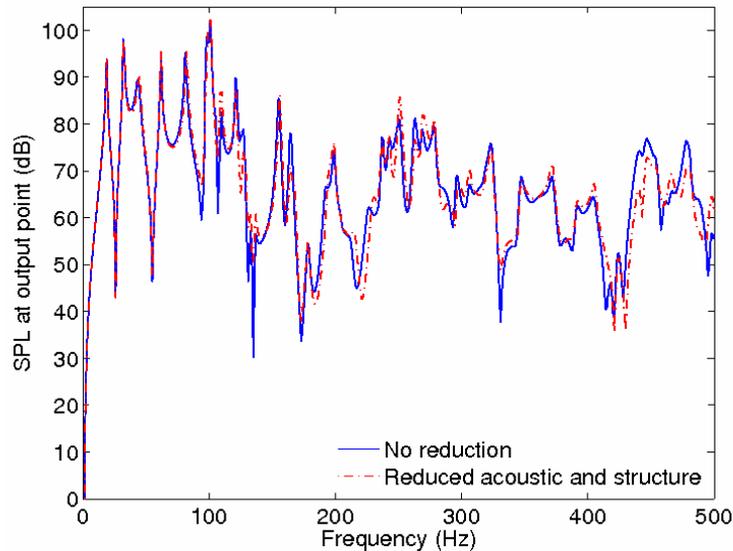


Figure 6: SPL FRF at output point: Modal reduction of acoustic and structural domains (1250 Hz and 1000Hz respective truncations).

Figure 6 presents the sound pressure level frequency response at the output point, comparing the precision reached by a modal-based reduced model to the reference finite element solution.

The acoustic cavity and the structural domain are dynamically condensed on its interface degrees of freedom with the structural domain and the impedance boundary surface. The output dof as well as the structural excited dof are kept uncondensed so that no back-substitution step is required.

The results are presented for a truncation at 1250 Hz (Cf. Previous section) for the acoustic domain, and 1000 Hz for the structural domain. Non-significant CPU time improvement is observed due to the little amount of structural dofs to be condensed.

Furthermore, the structural mesh does not enable a proper representation of the mode shapes up to 1000 Hz. Therefore, the results presented on Figure 6 exhibit a fairly average approximation of the complete problem, with noticeably hindered precision above 400 Hz.

Figure 7, plotting the mean quadratic pressure in the acoustic cavity for the same reduction conditions exhibits a satisfactory approximation up to 350 Hz.

Above this point, the solution substantially deteriorates with an increased modal density.

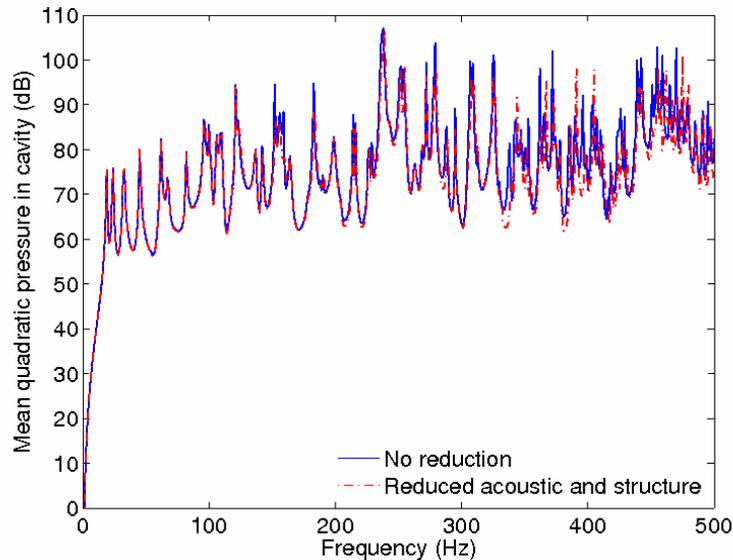


Figure 7: Mean quadratic pressure FRF (cavity): Modal reduction of acoustic and structural domains (1250 Hz and 1000Hz respective truncations).

d) Conclusion:

(Strength points and weakness of the method, Are the results good over all the frequency range?, ...)

The use of a modal approach brings substantial computational improvements for the calculation of frequency responses by a frequency sweep, using the finite element method. The reduction in size of the problem to solve at each frequency increment, keeping a good approximation of the original problem, makes it an interesting tool to improve the cost of an FE approach over an extended frequency range.

Even though tested on a fairly simple geometry where other methods would be much more efficient, its advantage lies in its performance over a wide range of situations. Furthermore, the component approach allows to work independently on different parts of a complex system, which can prove useful for design purposes.

However, as illustrated on Figures 6 and 7, the extension toward the mid-frequency region requires an even refined mesh in order to attempt the use of a modal approach, and becomes unadapted when a too high modal density is reached.

(Put in attachment: *.xls file with data table and chart of the results)